FLUID FLOW ANALYSIS FROM IMAGE SEQUENCES

Comunicación efectuada por la Dra. Etienne Mémin
en la sesión privada extraordinaria
de la Academia Nacional de Ciencias de Buenos Aires
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Abstract

In this paper, we present different methods allowing the reconstruction of the apparent motion field of a fluid flow from a time resolved image sequence. The techniques considered here involve specific data models built from physical consideration and adapted spatial or dynamical priors. In order to impose a dynamical consistency of the estimates, the dynamical constraints are embedded in a stochastic filtering framework or within an optimal control strategy. Both techniques enable to reconstruct the complete trajectory of motion field from noisy and incomplete measurements.

Key-words: fluid motion analysis, stochastic filtering, optimal control, data assimilation.

1. Introduction

The analysis and control of complex fluid flows is a major scientific issue in numerous application domains ranging from experimental visualization in fluid mechanics to geophysical flow analysis in environmental sciences. In particular, accurate measurement of atmospheric flow dynamics is of the greatest importance for weather forecasting, climate prediction or analysis. In that prospect flow visualization and the accurate extraction of flow velocity fields or of any dynamical measurements are of the utmost importance. Recently, the study of dynamical structures and the estimation of dense velocity fields from fluid image sequences have received a great attention from the computer vision community. These works depart somewhat from the studies conducted within the experimental fluid mechanics community, which has developed efficient correlation techniques called Particles Image Velocimetry (PIV) for the estimation of flow velocities of fluid seeded with particles (dye, smoke, particles) and imaged on a laser light sheet (see Raffel *et al.* 2007, for a recent review on these methods). An underlying assumption of PIV technique is that the motion of these tracers follows the motion of the neighboring fluid particles. This condition is not always satisfied and requires seeding the flow with small sized tracers leading to an increase in measurement difficulties. Some phenomena, such as natural convection or momentum transfer between the particles and the fluid, may be influenced by the large amount of seeding particles and the seeding may, in return, alter results. Moreover, the experimental setup implied by the adjustment of seeding, and the tuning of the optical parameters according to the flow dynamics and the targeted observation scale makes the implementation rather tricky, if not tedious. Consequently, these techniques are mainly restricted to small closed loops wind tunnels (typically a square section of 15 cm). Although those techniques have the advantage to limit local errors propagation their local nature provides only sparse measurements and prevent the introduction of physical laws within the estimation process to guarantee a temporal or spatial consistency of the motion fields. This is all the more true in meteorology where clouds are used as tracers for recovering atmospheric wind vectors from correlation techniques. The wind fields routinely extracted with such techniques are usually very sparse. Such sparse motion fields are not sufficient for capturing complex fluid flow phenomena like turbulent structures, which develop at a wide range of scales simultaneously.

The analysis of motion of general scalar and time resolved image sequences depicting fluid flows such as satellite or passive scalar images are particularly challenging due to abrupt and sudden changes of the luminance function. For these reasons, motion analysis techniques designed for computer vision application and quasi-rigid motions, are not well adapted in this context. Recently, methods for fluid-dedicated dense estimation have been proposed to characterize fluid motion (Corpetti *et al.*, 2002; Corpetti *et al.*, 2003; Cuzol *et al.* 2007; Ruhnau *et al.* 2006, Ruhnau *et. al*, 2007; Yuan *et al.*, 2007). These estimators have been applied to different application domains and have shown to provide accurate measurements. The general principles of such techniques and how they can be adapted to different application contexts are presented in section 2. These motion estimators rely nevertheless on a small set of images and may thus suffer from a temporal inconsistency when they are applied sequentially on time resolved image sequences. In that case, the set of motion fields provided may not respect fluid mechanics conservation laws. The design of appropriate approaches enabling to take into account the underlying physics of the observed flow constitutes a widely open domain of research. In section 3 and 4, we will describe several techniques proposed recently and that enable a consistent estimation of the fluid motion velocity fields.

2. Fluid flow velocity estimation

The motion estimation problem consists to estimate from an image sequence *I(x,t)* a vectorial function $\Omega \subset R^2 \rightarrow v(x) = (u_x(x), u_y(x))^T$ defined in the image domain Ω. In order to perform such an estimation, it is first assumed that the unknown motion is directly related to the spatial and temporal variations of the intensity function. The unknown motion field is also supposed to be to some extent spatially smooth. Two different frameworks are then usually considered for such an estimation: *discrete correlation approaches* and *differential optical flow techniques*.

2.1. Correlation techniques

Correlation methods are based on a local minimization strategy. They aim at recovering at a given point the velocity vector that minimizes over a localized search window and a discrete state space a cost function encoding the disparity between the image at time *t* and the displaced image at time *t+1:*

$$\sum_{r \in W(x)} C(I\ (r+v, t+1), I\ (r,t)). \tag{1}$$

The cost functions considered are usually the squared difference or the correlation function. These techniques can be very efficiently implemented in the Fourier domain when the correlation function is considered. As discrete techniques, they are however prone to an erroneous spatial variability. Specific contexts such as the observation of flow seeded with particles have enabled the design of efficient correlation techniques for the measurement of the large scales of the flow (Adrian, 1981; Adrian, 2005; Raffel *et al.* 2007). However, due to their local nature, correlation techniques provide reliable measurements only on localized region for which the brightness pattern can be unambiguously identified. They allow as a consequence only sparse measurements and require the use of additional post-processing technique to compute spatial derivatives of the velocity field. The sparsity, and the discrete nature of the velocity field prevent from incorporating within this framework any continuous physical constraint which limits considerably their evolution toward an accurate and reliable flow reconstruction.

2.2. Differential optical-flow

Differential methods are formulated as the minimization of a continuous functional defined on the whole image space (Horn and Schunck, 1981). The functional is generally composed of two terms: (*i*) a data model relating the unknown velocity fields and the observed luminance variations and (*ii*) a smoothness constraint that enforces a spatial smoothness of the solution:

$$\int_{\Omega}(\frac{dI}{dt}+f(I,v))^{2}+\lambda(g(\partial^{n}u_{x},\partial^{n}u_{y}))dx. \quad (2)$$

These techniques are more general than correlation techniques as they allow the definition of dedicated data model and smoothing functions. The simplest model (Horn and Schunck, 1981) consists to rely on a brightness conservation assumption (f (I,v) =0) and on a first order smoothing penalizing the magnitude of the gradient of the velocity components. This model tends to over smooth the estimated vector field and like the correlation techniques, relies on a brightness variation model that does handle 3D motion effects. Recently several estimators dedicated to fluid flows have been proposed (Arnaud et al. 2006, Corpetti *et al.* 2002; Héas *et al.*, 2007-a; Héas *et al.*, 2008). These estimator are based on the one hand on smoothness functions that preserve compact blobs of divergence and curl and on the other hand, on specific data models built from the continuity equation and a relation between the luminance function and a state variable of the fluid. This kind of estimators allows introducing additional constraints such as a large scale dynamical prior (Héas *et al.*, 2007-a) or correlation based constraints in order to authorize long range displacements (Heitz *et al.* 2008). Another alternative consists to express directly the motion field as the combination of basis functions. Such a technique has been proposed in Cuzol *et. al*, (2008) for radial basis functions known as vortex particles and which are derived from a smoothed discretization of the Biot-Savart law (Chorin 1973; Leonard 1980; Cottet & Kumoutsakos, 2000). This latter estimator is well suited to a fast characterization of the large scales of the flow.

The differential techniques supply dense motion fields and allow theoretically accessing to finer scales of the motion. They are however relying only on a small number of consecutive images and the sequential application of the estimation process on an image sequence does not guaranty that the sequence of motion fields obey the flow conservation laws. In order to impose such a dynamical consistency it is necessary to consider the motion estimation problem as a tracking problem. This tracking can be either formulated into a stochastic filtering formulation or within an optimal control framework.

3. Recursive estimation through stochastic filtering

In order to estimate optimally the complete trajectory of an unknown state variable from a sequence of past image frames, we formulate the problem as a stochastic filtering problem. Resorting to stochastic filters consists in modeling the dynamical system to be tracked as an hidden Markov state process. The goal is to estimate the value of the random Markovian process denoted $X_{0:n} = \{X_t\}, t \in [0,n]$ from realizations of the observation process. The set of measurements operated at discrete instants are denoted $Z_{0:n} = \{Z_0,\ldots,Z_n\}$. The system is described by *(a)* the distribution of the state process at the initial time $P(X_0)$), *(b)* a probability distribution modeling the evolution of the state process $p(X_k \mid \Im_{t<k})$ where $\Im_{t<k}$ denotes the filtration generated by X_t until time *t* and *(c)* a likelihood (representing the *measurement equation*) $p(Z_k \mid X_k)$ that links the observation to the state variable. In this framework, the *posterior* distribution — *i.e.* the law of the state process knowing the whole history of observations — carries the whole information on the process to be estimated. More precisely, as tracking is a causal problem, the distribution of interest is the law of the state given the set of past and present observations $p(X_k \mid Z_{1:k})$ known as the *filtering distribution*. The problem of recursively estimating this distribution may be solved exactly through a Bayesian recursive solution, named the optimal filter (Gordon *et al.*, 1993). This solution requires computing integrals of huge dimension. In the case of linear Gaussian models, the Kalman filter (Kalmann, 1960; Kalman and Bucy, 1961) gives the optimal solution since the filtering distribution is in that case Gaussian. In the nonlinear case, an efficient approximation consists in resorting to sequential Monte Carlo techniques (Arulampalam *et al.*, Doucet *et. al.*, 2000; Gordon *et al.*, 1993). These methods consist in approximating the filtering distribution in terms of a finite weighted sum of Dirac masses centered in elements of the state space, named particles. At each discrete instant, the particles are displaced according

to a probability density function named *importance function* and the corresponding weights are updated using the system's equations. A relevant expression of this function for a given problem is essential to achieve an efficient and robust particle filter. Interested readers may found different possible choices in (Arnaud and Mémin 2007, Doucet *et al.* 2000).

Such a technique has been applied to the tracking of a solenoidal field described as a combination of vortex particles (Cuzol and Mémin, 2008). The motion field in that work is described through a set of random variables $x_i, i = 1,...,p$:

$$v(x) \approx \sum_{i=0}^{p} \gamma_i K_{\varepsilon_i}^{\perp}(x_i - x), \qquad (3)$$

where $K_{\varepsilon_i}^{\perp}$ is a smoothed Biot-Savart kernel obtained by convolving the orthogonal gradient of the Green kernel associated to the Laplacian operator with a smoothing radial function. The vector $x_t = (x_i(t), i = 1,...,p)^T$ represents the set of vortex particle locations and the coefficient, g_i, their strength. The dynamics of these random variables is defined through a stochastic interpretation of the vorticity transport equation:

$$dx_t = v(x_t)dt + \sigma dB, \qquad (4)$$

where dB stands for a $2p$-dimensional Brownian motion with independent components, and whose diffusion coefficient is defined from the fluid viscosity. The evolution of the vortex set, between two frame instants is represented by the following Markov transition equation:

$$p(x_i(t) \mid x_i(t - \Delta t)) \propto N(x_i(t - \Delta t) + v(x_t), \sigma^2 \Delta t I_{2p}), \qquad (5)$$

where I_{2p} denotes the $2p \times 2p$ identity matrix. A sample of the trajectories generated between two frames is then weighted according to the likelihood $p(z_k | x_k)$. In this work, this density has been defined in terms of a reconstruction error measurement, computed from the pair of images (I_k, I_{k-1}). Different results obtained for the tracking of airplane tip wing's vortex or cyclones can be found in (Cuzol & Mémin, 2008).

4. Optimal control for dynamically consistent estimation

In this section, we present the second alternative for a dynamical filtering of noisy and incomplete data. This framework ensues from control theory and has been popularized in geophysical sciences where it is known as *variational assimilation* (Le-Dimet & Talagrand, 1986; Lions, 1971). Contrary to particle filtering, variational assimilation techniques have the advantage to enable a natural handling of high-dimensional state spaces. Before presenting further the adaptation of such a framework to motion estimation, let us describe the general notions involved.

As previously, the problem we are dealing with consists in recovering a system's state $X(x, t)$ obeying a dynamical law, given some noisy and possibly incomplete measurements of the state. The measurements, in this context also called observations, are assumed to be available only at discrete points in time. This is formalized, for any location, *x*, at time $t \in [t_0, t_f]$ by the system:

$$\begin{aligned} &\frac{dX}{dt} + M(X(x,t), c(t)) = 0, \\ &X(x, t_0) = X_0 + \eta, \end{aligned} \tag{6}$$

where M is a non-linear dynamical operator depending on a control parameter $c(t)$. The term X_0 is the initial vector at time t_0, and h is an (unknown) additive control variable of the initial condition. Furthermore, we assume that measurements of the unknown state, Y, are available. These observations are measured through the non-linear operator, H. The objective consists then to find an optimal control of low energy that leads to the lowest discrepancy between the measurements and the state variable. This leads to the minimization of the following cost function:

$$J(c,\varepsilon) = \frac{1}{2}\int_{t_0}^{t_f} \| Y - H(X(c(t),\varepsilon,t)) \|^2_{R^{-1}} dt + \frac{1}{2} \| \varepsilon \|^2_{B^{-1}} + \frac{1}{2}\int_{t_0}^{t_f} \| c(t) - c_0 \|^2_{F^{-1}} dt, \tag{7}$$

where c_0 is some expected value of the control parameters. The norm $\|\bullet\|^2_{A^{-1}}$ is induced by the inner products $\langle A^{-1} \bullet, \bullet \rangle$, and *R*, *B*, *F* are covariance matrices of the observation space and state space. They are respectively related to the observations, the initial condition of the state variable and to the expected value of the control variable. Regarding the minimization of the objective function, a direct numerical evaluation of the functional gradient is computationally infeasible, because this would require to compute perturbations of the state variables along all the components of the control variables $(c(t), e)$ — *i.e* to integrate the dynamical model for all perturbed components of the control variable, which is obviously not possible in practice.

A solution to this problem consists to rely on an *adjoint formulation* (Le-Dimet and Talagrand, 1986; Lions, 1971). Through this formalism, the gradient functional is obtained by a forward integration of the dynamical system followed by a backward integration of an adjoint dynamical model. This model is defined by the adjoint of the discrete scheme associated to the dynamical system.

Such a technique has been recently applied to the estimation of fluid motion fields (Corpetti *et al.*, 2007; Héas *et al.*, 2007-b; Papadakis and Mémin, 2008; Papadakis *et al.* 2007) , and to the tracking of closed curves (Papadakis and Mémin, 2008). These works rely either on a shallow water dynamical model or on a vorticity-velocity formulation. They associate motion measurements given by external motion estimators (Papadakis and Mémin, 2008) or incorporate directly luminance data (Papadakis *et al.* 2007). The first case provides a denoising technique that allows improving significantly the observed motion fields. The second technique constitutes a complete autonomous motion estimator that enforces dynamical coherence and a temporal continuous trajectory of the solution. These approaches, compared to traditional motion estimator, enable to recover accurately a broad range of motion scales. A similar technique has been also used recently to recover the parameters of a reduced dynamical system obtained from a POD-Galerkin technique (D'Adamo *et al.* 2007). This technique improves significantly the accuracy of the estimated reduced system. For a flow showing periodic behavior this method allows to restore noisy experimental velocity fields provided by standard PIV techniques and to reconstruct a continuous trajectory of motion fields from discrete and unstable measurements. Experimental results on satellite meteorological images, on 2D turbulent flows and on wakes flows can be found in the different aforementioned publications.

5. Conclusion

In this paper we have briefly presented several motion estimators dedicated to the measurement of dense velocity fields from image sequences. These techniques incorporate physical constraints within the data model on which they are relying or through dynamical priors. These latter constraints have been embedded in a stochastic filtering framework or within an optimal control strategy. Both solutions enable to reconstruct a coherent trajectory along time of the fluid flow velocity fields.

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IRISA/Univ. Rennes I, Campus Beaulieu,
35042 Rennes Cedex, France
memin@irisa.fr